Marcus Löfverström

Laboration instructions

Barotropic Instability

September 2011

1 Objective

The purpose of this laboration is to test the theory of barotropic instability in a number of semi-real hands-on computer experiments. The first part of the laboration considers barotropic instability from a theoretical point of view and the students are to derive Rayleigh's equation and Rayleigh's criterion from the quasi-geostrophic potential vorticity equation. The second part considers experimental work using a numerical quasi-geostrophic model where the theory is verified.

The model used in the laboration was originally written by Anders Engström and Sebastian Mårtensson as a project in the NGSSC course (National Graduate School of Scientific Computing) *Programming in Science and Technology*. They created the core of the model, *Harmony*, which is used for sending variables between gridpoint and spectral space. The model dynamics was validated by solving the *shallow water equations*. The quasi-geostrophic code used here was written by Marcus Löfverström as a part of his degree project.

2 Theory

The characteristics of a hydrodynamic instability is dependent on both the vertical and horizontal structure of the stratified background flow. In a baroclinic atmosphere the potential density is a function of both pressure and potential temperature and the geostrophic wind has, in general, a vertical shear. Baroclinic instability is one of the major players in the atmospheric system and it is responsible for cyclogenesis, a very important feature that is further discussed elsewhere, see e.g. Holton (2004).

Barotropic instability, on the other hand, occurs in regions with strong horizontal shear such as jet streams. In a barotropic atmosphere the potential density is only a function of pressure meaning that isopycnic surfaces (isosurfaces for potential density) are also isobaric surfaces and that isothermal and isopycnic surfaces are parallel thus $\nabla_p T = 0$ and the thermal wind relation becomes $\partial \mathbf{v}_g / \partial p = 0$. The thermal wind describes the height dependence of the geostrophic wind, which, in this case, is only dependent on the horizontal position and time.

The laboration aims to examine a horizontal shear-flow instability occuring in an idealized barotropic jet on a sphere. The instability is in many regards similar to *Kelvin-Helmholtz instability*, see Fig. (1), except that the latter also needs a large density shear, cf. Vallis (2006), something that is not present in this study.

The quasi-geostrophic potential vorticity is here simplified as;

$$q \equiv \nabla^2 \psi + \beta y, \tag{1}$$

and the quasi-geostrophic potential vorticity equation for a two dimensional Boussinesq flow reads;

$$\frac{\partial q}{\partial t} + \mathbf{v}_g \cdot \nabla q = \frac{\partial}{\partial t}\nabla^2\psi + (\hat{k} \times \nabla\psi) \cdot \nabla\left(\nabla^2\psi\right) + \beta\frac{\partial \psi}{\partial x} = 0. \tag{2}$$

In the equations above ψ is a streamfunction and $\nabla^2\psi$ defines the relative vorticity. The geostrophic wind expressed in the streamfunction is $\mathbf{v}_g \equiv \hat{k} \times \nabla\psi$ ($u_g \equiv -\frac{\partial\psi}{\partial y}$ and $v_g \equiv \frac{\partial\psi}{\partial x}$). By assuming wave solutions in Eqn. (2), *Rayleigh's equation* can be derived;

$$(U-c)\left(\frac{\partial^2\widetilde{\psi}}{\partial y^2} - k^2\widetilde{\psi}\right) + \left(\beta - \frac{\partial^2 U}{\partial y^2}\right)\widetilde{\psi} = 0. \tag{3}$$

From Eqn. (3) *Rayleigh's criterion* can be derived that describes conditions for barotropic instability.


(a) (b)

Figure 1: Kelvin-Helmholtz instability seen in clouds. Images are taken from *http://cloudappreciationsociety.org/*

3 Model description

The numerical model contains a spherical harmonic transform module called *Harmony*, which transforms variables between gridpoint space and spectral space. The spectral transforms are borrowed from *ECMWF* and the Gaussian latitudes from *PUMA* (Portable University Model of the Atmosphere) written at University of Hamburg.

In global spectral models the horizontal representation of dynamic fields is based on orthogonal spherical harmonic functions, which are eigenfunctions of the Laplace operator on a sphere. Variables are projected on truncated series of associated Legendre polynomials, P_n^m, in the meridional direction and Fourier series, F_n^m, in the zonal direction. The associated Legendre polynomial of order m and degree n is defined as;

$$P_n^m(x) = \left[(2n+1)\frac{(n-m)!}{(n+m)!}\right]^{\frac{1}{2}} \frac{(1-x^2)^{\frac{m}{2}}}{2^n n!} \frac{d^{n+m}}{dx^{n+m}}(x^2-1)^n, \tag{4}$$

where $x = sin\,\theta$, and θ is the latitude. The spectral representation of an arbitrary variable Y is;

$$Y(\lambda,\theta) = \sum_{m=0}^{M}\sum_{n=m}^{M} \hat{Y}_n^m P_n^m(sin\,\theta)e^{im\lambda}, \tag{5}$$

where λ is the longitude and $\hat{Y}_n^m$ are the spectral coefficients of the variable Y. The benefit with the spectral representation is that all horizontal derivatives become trivial. For example, the laplacian of a variable Y is simply;

$$\nabla^2 Y \rightarrow -\frac{n(n+1)}{a^2}\hat{Y}_n^m,$$

where a is the radius of the Earth and n is a wave number.

Since all variables are represented by waves, both in the zonal and meridional direction, phenomena smaller than the shortest wave length can not be resolved. The highest possible wave number that can be resolved (shortest wave), called the truncation wave number (k_{trunc}), is calculated from the grid resolution by;

$$k_{trunc} = \frac{g_\lambda - 1}{3} = \frac{2 \cdot g_\theta - 1}{3},$$

where g_λ and g_θ are the number of gridpoints in the zonal and meridional direction respectively. Normal resolutions for climate models are $g_\lambda \times g_\theta = 128 \times 64$ and 64×32 and the truncation wave numbers are $k_{trunc} = 42$ and 21 respectively, thus the resolutions are usually referred to as T42 and T21.

In the model, an idealized Gaussian jet stream on the Northern hemisphere is represented by

$$u = u_0 e^{-\left(\frac{\theta-\theta_0+\varepsilon sin(\lambda)}{b}\right)^2}, \tag{6}$$

where u_0 is the maximum wind speed, θ_0 a reference latitude and b is a scaling factor that controls the width of the jet. Due to the physics of the problem the jet needs to be perturbed slightly for instabilities to occur. This is provided by ε that is the amplitude of a sine wave in the zonal direction that makes one period around a latitude circle.

4 Theoretical exercises

Exercise 1

Derive *Rayleigh's equation* (3), from the quasi-geostrophic potential vorticity equation (2), defined in the *Theory* section.

Exercise 2

Derive *Rayleigh's criterion* for instability from *Rayleigh's equation.* Also, discuss what it means and how it can be used to determine stability/instability.

Exercise 3

Why is it necessary to perturb the jet for instabilities to occur?

5 Laboratory exercises

Get familiar with the code and learn to run it on the computer. Default values of wind speed is $u_0 = 30\ ms^{-1}$ and width of the jet is $b = 10$ gridpoints. This is a stable eastward jet. Start by increasing the wind speed to, for example, 100 ms^{-1} and examine the difference to the behaviour in time of weaker jets. The stronger jet is unstable which is seen as vortices that are emerging after a while. In the stable jet vortices should not be created. Find the limit where the jet becomes unstable. Hint; plot the meridional potential vorticity profile.

After changing the wind speed you can try to change the width of the jet. This parameter is a bit sensitive to changes so keep it close to its default value. A narrow jet becomes unstable more easily than a wider jet.

Lastly, change the direction of the jet. This is done by setting u_0 to a negative value. Examine the conditions for stability and difference in behaviour from the eastward jets.

Analyze the stability with help of Rayleigh's criterion.

Examination

You can either work alone or in groups of two or maximum three people. The examination is a short individually written report including a description of the experiments, results and conclusions. Present the results in text with tables and images as a help for the reader. Remember to only present relevant results and do not swamp the report with images. All figures in the report must be referred to in the text! Present a few illustrative examples and discuss what they show and how they are interpreted physically. Sloppy descriptions and interpretations will not be accepted!

6 Lab execution

The model is executed from a terminal window found under *start/utilities/terminal*. Follow these steps to copy the program files to your own work-space. A few useful terminal commands are presented in the next section of the instruction.

1. Login to MISUs computer system.

2. Create a directory/folder to copy the necessary Fortran and Matlab files to. The following command lines are used to go to the desktop and create a directory called “gfd” (you can create it wherever you like and call it whatever you please, this is just a tutorial of the basic commands)

   ```
   cd Desktop
   mkdir gfd
   ```

3. To copy the required program files to *gfd* use the commands

   ```
   cd gfd
   cp -r /afs/su.se/domain/misu.su.se/lab/gfd/* .
   ```

   You are now copying the contents of the gfd-folder plus all subdirectories to your local *gfd*-directory. The second argument (.) tells the computer to put the copied files in the current directory, in this case in *gfd*.

4. Start Matlab and type

   ```
   addpath '/afs/su.se/home/a/b/abcd1234/Desktop/gfd/src'
   ```

   where *abcd1234* is your own login name. This gives Matlab access to the outfiles when running the model. If you have created the directory elsewhere, i.e. not at the Desktop, type “*pwd*” in the terminal window to get its full path.

The model is written in Fortran, which was one of the first high-level programming language when it was developed in the 1950-ties. It is a very fast language specialized for numerical calculations thus it is commonly used in advanced modelling (all numerical weather prediction models and climate models are written in Fortran). When using Fortran, the code has to be compiled before it can be executed. This is done with a compiler, e.g. *gfortran* or *ifort*, which reads the code and writes an executable binary file. The compilation of this model is done by the script *Makefile*. Simply type “make” in the terminal window (when standing in the *src*-folder) to execute the compilation script. This has to be done every time a change is made in the source code. To execute the model, type “./quasi.x”. All outfiles end up in the *src-directory* and they can be analyzed with e.g. Matlab. A plot script is provided, called *plot_fields.m*.

A summary of the procedure of running the model and analyzing the results is thus

1. Go to the src-directory in the terminal window.
2. Type “make” to compile the model
3. Type “./quasi.x” to run the executable file
4. Analyze the results in Matlab with “*plot_fields.m*”

A few more comments about the code. All files containing source code have the suffix .f90. There are three .f90-files in the model, *quasi.f90*, *fft.f90* and *harmony.f90*. The first one, *quasi.f90*, is the main program for the laboration.

It contains the quasi-geostrophic code and uses routines from *fft.f90* which performs Fourier transformations and *harmony.f90* where the Legendre transformations are done.

In Fortran one needs to define all variables (and their right dimensions) before they can be used. All variables used by quasi.f90 are defined in *vars.h.* Remember to add the variable to *vars.h* if you add something to the code. There is also a file called *parameters.h* that contains other important information about the model such as grid resolution and number of time-steps. When adding code to any of the .f90 or .h files the model needs to be recompiled.

7 Useful terminal commands and “emacs”

Terminal commands

This is a list of some useful terminal commands that can come in handy when navigating in a terminal environment. Take it slow and think before you act, it can be hard to salvage accidentally deleted files.

cd - “change directory” is used to navigate between folders. Type “cd” followed by the name of the directory you want to go to. To go one step up type *“cd ..”* and two steps up type *“cd ../..”*

pwd - “print working directory” gives the full path to the directory where you are standing

ls - “list” content of a folder. Can be used with *“ls -l”* to get more details

“tab” - “command completion”. A very helpful tool to utilize when writing long names or paths. An example, when writing */afs/su.se/domain/misu.su.se/lab/gfd/* (the third paragraph in the *Lab execution*) you can type */a[tab]* → */afs/*, type */afs/su[tab]* → */afs/su.se/*. The system fills in the rest of the path if there is only one option that fits your request. When there are multiple options the computer writes them all in the terminal window if [tab] is pressed twice. You can test this at */afs/su.se/domain/mi[tab][tab]* where *miljo.su.se* and *misu.su.se* should show up in the terminal window.

cp file/directory - “copy” a file or directory. Use it as *“cp file /path-to-new-home-of-the-file”*. When copying a directory use *“cp -r directory /path-to-new-home-of-the-directory”* to also copy all subdirectories.

rm file - “remove” a file. To delete a whole directory with subdirectories type *“rm -r”* followed by the directory-name.

mkdir name - “make directory” creates a directory. To create a directory called *my-directory* type *“mkdir my-directory”* *mv file/directory* - "move or rename" *"mv name1 name2"* rename name1 to name2. Can be used both on files and directories. To move the file/directory called *name1* type *"mv name1 /path-to-new-home-of-the-directory/name1"*

* - a wild card that can mean any string. Example, if you want to copy many files with the same suffix (e.g. .f90) simply type *"cp *.f90 /path-to-new-home-of-the-files"*.

Emacs

It is recommended to use an editor called *emacs* to open .f90 files. To open a file type

emacs filename.f90 &

The ampersand at the end (&) allows you to use the terminal window when *filename.f90* is open. To save changes in an emacs script hold down *ctrl* and press *x s*. When closing emacs hold down *ctrl* and press *x c*.

References

[Holton (2004)] Holton, J. R. 2004 An introduction to dynamic meteorology *Elsevier Academic Press* 532 pp

[Vallis (2006)] Vallis, G. K. 2006 Atmosphere and Oceanic Fluid Dynamics Fundamental and Large-Scale Circulation *Cambridge University Press* 745 pp