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Notes and Correspondence
Is the Coriolis effect an 'optical illusion'?

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The difference between the derivations of the Coriolis effect on a rotating turntable and on the rotating Earth is discussed. In the latter case a real force, the component of the earth's gravitational attraction, non-parallel to the local vertical, plays a central role by balancing the centrifugal force. That a real force is involved leaves open, not only the question on the inertial nature of the 'inertial oscillations', but also the way we tend to physically conceptualize the terrestrial Coriolis effect.

Key Words: Coriolis effect; inertia oscillations; dynamic meteorology; conceptual models

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> 'The Coriolis Effect isn't a force so much as an optical illusion, brought about because we forget that we, too, are spinning with the ground beneath our feet.'(Walker, 2007, p. 124).

1. Introduction

In the introduction to his 'Theoretical Concepts in Physics', Malcolm S. Longair, Jacksonian Professor in natural philosophy at University of Cambridge, discusses the development of intuitive, physical insight. Although physical intuition on one hand is the source of many of the greatest discoveries in physics through 'leaps of imagination', it can also be a 'dangerous tool' because it can lead to 'some very bad blunders' (Longair, 1994, pp. 3 and 10). To avoid 'blunders', intuitive or heuristic conceptual models of physical processes must be in agreement with the underlying mathematics or at least not contradict it.

In the meteorological field there are examples of misfits between the mathematics and conceptual models (Persson, 2010). This is particularly true with respect to the terrestrial Coriolis effect in the atmosphere and oceans. Several textbooks, both popular and academic, often reason along the lines that 'relative to a rotating frame, such as that of the Earth, a fluid element may appear to be changing its direction of motion when relative to an inertial frame it is not'(White, 2003, p. 695). Since Newton's second law does not apply in a rotating frame of reference, the Coriolis force is then introduced as a convenience, as a 'mental construct' designed 'to make it appear' that Newton's second law is still valid (James, 1994, p. 8). Others see it as a 'semantic trick' that proves 'useful' for producing a 'notationally economical format' of equations of motion (Stommel and Moore, 1989, p. 72).

This Note will not only question the pedagogical value of such statements, but also their relevance for a scientific understanding of how the deflective mechanism due to the Earth's rotation affects the motions of the atmosphere and oceans.

2. The standard derivation of the Coriolis force

It is true that the standard derivation of the Coriolis effect through a coordinate transformation seems to support the notion that we are dealing with an 'optical illusion': an observer in a fixed (f) frame of reference sees an object move over time (t) without friction with absolute velocity ($\mathbf{V}_\mathrm{f}$) rectilinearly with zero acceleration if no forces are applied

$$\frac{\mathrm{d}\mathbf{V}_\mathrm{f}}{\mathrm{d}t} = \mathbf{0}, \tag{1}$$

whereas an observer in a rotating ($\boldsymbol{\Omega}$), relative frame of reference (r), sees the object move with relative velocity ($\mathbf{V}_\mathrm{r}$) in a curved, outward trajectory determined by the acceleration

$$\frac{\mathrm{d}\mathbf{V}_\mathrm{r}}{\mathrm{d}t} = \frac{\mathrm{d}\mathbf{V}_\mathrm{f}}{\mathrm{d}t} - \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{R}) - 2\boldsymbol{\Omega} \times \mathbf{V}_\mathrm{r} \tag{2}$$

where **R** denotes the distance from the centre of rotation (Holton, 2004, p. 33; Vallis, 2006, 2010, p. 53; Holton and Hakim, 2012, p. 36). The terms on the right-hand side are, *per unit mass*, the centrifugal force and the force named after one of its discoverers, Gaspard Gustaf Coriolis (1794–1843).

Equation (2) quite accurately describes an object's relative motion without friction over a rotating turntable; an observer on the turntable will see it move away in an ever widening Archimedean spiral with *increasing* speed. However, an observer on the rotating Earth following an object similarly moving without friction will see something quite different: how it performs more or less complicated 'inertial oscillations' within a confined space with *constant* speed. This applies, e.g. to drifting oceanic buoys, icebergs and floating debris.

Whereas the first observer, stepping off the turntable and into the absolute frame of reference, will see the object move rectilinearly according to Eq. (1), the second observer, hypothetically positioned in space, will see the object follow some curved (curtate cycloidal) trajectory on the Earth (Figure 1).

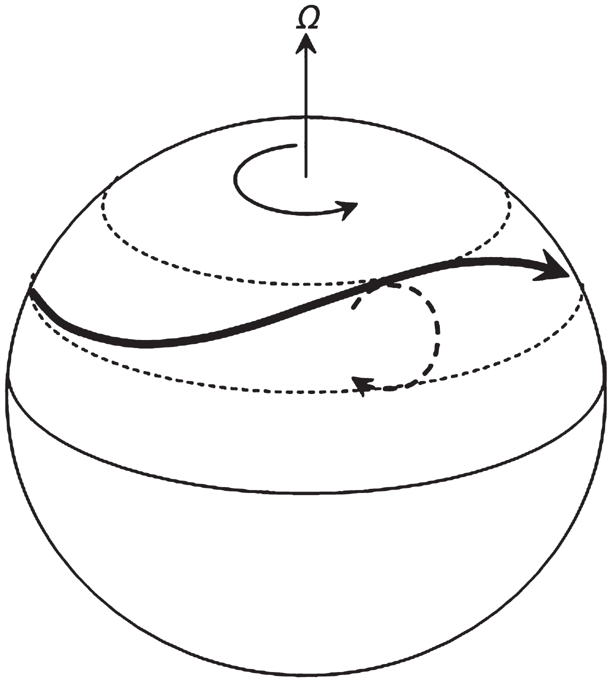

Figure 1. The relative motion of an object moving without friction in the midlatitudes follows a so-called 'inertia oscillation' (dashed line), while the absolute motion follows a curtate cycloid trajectory (full line).

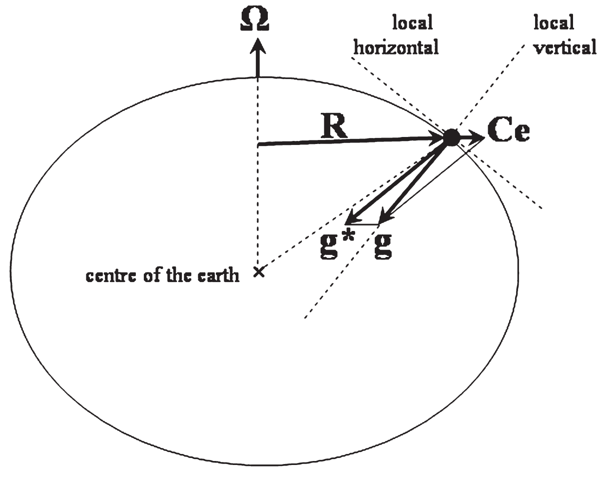

Figure 2. A schematic image of the non-spherical Earth with the gravitational vector **g*** (true gravity) and the centrifugal force **Ce** due to the Earth's rotation. Their resultant **g** (apparent gravity) is parallel to the local vertical and perpendicular to the local horizontal (dashed lines). Although it will not change the mathematics it is interesting to note that, due to the attraction of the equatorial bulge**,** **g*** is not directed exactly to the centre of the Earth (except at the Equator and the poles). Only if the total mass is concentrated in the centre of the Earth would **g*** be directed to that point (Phillips, 1973, 2000, figure 1).

Although the motions over the turntable, with a light-hearted simplification, can be compared with an 'optical illusion', this is obviously not true for motions over the rotating Earth.

3. The equations of motion on a rotating Earth and on a rotating turntable

For an object moving without friction over the Earth's surface Eq. (2) takes the form

$$\frac{\mathrm{d}\mathbf{V}_\mathrm{r}}{\mathrm{d}t} = \mathbf{g}^* - \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{R}) - 2\mathbf{\Omega} \times \mathbf{V}_\mathrm{r}, \tag{3a}$$

where **g*** is the Earth's gravitational attraction, Newtonian gravity or 'true gravity'. Similarly the path of an object moving without friction over a rotating turntable is determined by

$$\frac{\mathrm{d}\mathbf{V}_\mathrm{r}}{\mathrm{d}t} = \mathbf{g} - \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{R}) - 2\mathbf{\Omega} \times \mathbf{V}_\mathrm{r}, \tag{3b}$$

where **g**, the (apparent) gravity, keeps the object on the platform.

It is a source of semantic, and perhaps also scientific confusion, that in colloquial English, in contrast to, e.g. German, Dutch and the Scandinavian languages, the word 'gravity' can intend both **g*** and **g**. As the difference turns out to be fundamental, for the discussion this difference needs to be clarified (Holton, 2004, p. 14; Vallis, 2010, pp. 55 f; Holton and Hakim, 2012, p. 12 f). Due to its non-spherical shape, which the Earth has acquired due to its rotation, gravitation, or 'true gravity', **g*** is parallel to the local vertical only at the Equator and the poles. Gravity, or 'apparent gravity' (**g**), the resultant between the gravitational force and the centrifugal force

$$\mathbf{g} = \mathbf{g}^* - \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{R}) = \mathbf{g}* + \Omega^2\mathbf{R} \tag{4}$$

is everywhere parallel to the local vertical (Figure 2).

The apparent gravity has been introduced in Eq. (3b), although it has no horizontal component, to emphasize the almost identical mathematical structure between Eqs (3a) and (3b). In spite of their strong similarities the equations nevertheless, as noted above, yield quite different motions, both in a relative and an absolute frame of reference. The deflective mechanism through the Coriolis term $-2\mathbf{\Omega} \times \mathbf{V}_\mathrm{r}$ is the same for both equations, so the difference between **g** and **g*** appear as the sole source of the discrepancy.

4. The Coriolis force on a rotating planet

The resultant of Eqs (3a) and (4) are combined into

$$\frac{\mathrm{d}\mathbf{V}_\mathrm{r}}{\mathrm{d}t} = \mathbf{g} - 2\mathbf{\Omega} \times \mathbf{V}_\mathrm{r}, \tag{5}$$

where the second term through the cross-products implies that the deflection is to the right and always perpendicular to the motion and therefore can neither increase nor decrease the speed and thus its kinetic energy. As **g** has (by definition) no horizontal components, Eq. (5) will yield the familiar expression for the acceleration of horizontal motion (H) without friction over the Earth' surface at latitude ϕ

$$\left(\frac{\mathrm{d}\mathbf{V}_\mathrm{r}}{\mathrm{d}t}\right)_\mathrm{H} = f\mathbf{k} \times \mathbf{V}_\mathrm{r} \tag{6}$$

where **k** is the vertical unit vector and $f = 2\Omega \sin\phi$ is the Coriolis parameter (Holton, 2004, p. 13; Holton and Hakim, 2012, p. 12). As the deflection is perpendicular to the motion it will drive it into a curved path, which, to the extent the Coriolis force variation with latitude can be neglected, will yield circular trajectories, 'inertia circles'.

5. The trajectories of 'inertia oscillations'

More generally the trajectories satisfying Eq. (6) define so-called 'inertia oscillations', which have a radius of curvature, (ρ)

$$\rho = \frac{\mathbf{V}_\mathrm{r}}{2\Omega \sin\varphi} \tag{7}$$

which at midlatitudes, in a relative frame of reference, is associated with near circular trajectories, and at low latitudes symmetric loops around the Equator, both westward migrating for geophysical relevant velocities (Figure 3).

The two types of inertia oscillations can easily be understood in two ways. From Eq. (7) they can *kinematically** be seen

*'Kinematics' is the branch of mechanics where motions are considered without forces, in contrast to 'statics', where forces are studied without motion, and 'dynamics', which includes both motion and forces. Kinematics was developed during the 1800s as a useful tool in fluid mechanics (Persson, 1998, p. 1379 f).

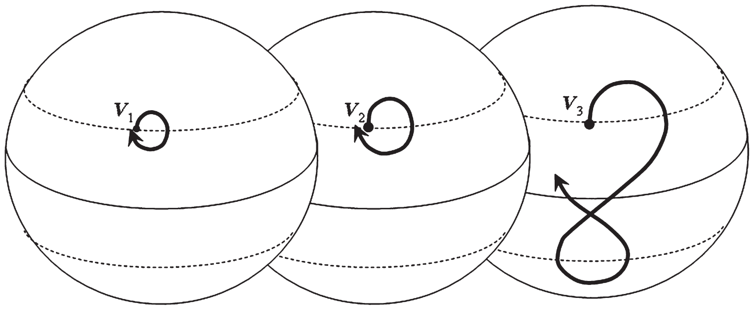

Figure 3. Trajectories relative to the rotating Earth of three objects, all starting from the same latitude, moving without friction over the rotating Earth's surface with different relative speeds ($V_1 < V_2 < V_3$) performing inertia oscillations. For moderate speeds at higher latitudes the motion will follow approximate circular trajectories, 'inertia circles'. For higher speeds at lower latitudes the trajectories will be symmetric around the Equator (after Ripa, 1997a, figure 2).

as an interplay between relative velocity V_r and decreasing curvature with approach towards lower latitudes (Paldor and Killworth, 1988; Ripa, 1997a), *dynamically* as an interplay between angular momentum conservation and conservation of energy, as a 'contest' between the equatorward centrifugal force and the poleward attraction of gravitation (Brooks, 1948; Feussner, 1969; Paldor and Killworth, 1988; Ripa, 1997a; Phillips, 2000).

Paldor and Killworth (1988) derived an expression for the initial latitude φ_0 poleward from which an eastward velocity u_0 will follow an 'inertia circle' trajectory and not a symmetrical 'loop' around the Equator.

$$\cos\varphi_0 < 1 - \frac{2u_0}{2\Omega R} \tag{8}$$

For velocities of 10 m s^{-1} this yields latitude 12°, which approximately encloses the equatorial Tropics with its particular dynamics. For velocities of 50 m s^{-1} it yields a latitude of almost 27°, just equatorward of the average position of the subtropical jet stream. See Paldo and Killworth, 1988 for full discussion and more complex scenarios.

The *absolute* motion of objects taking part in these oscillations, seen by the hypothetical observer outside the Earth, will obviously not follow straight lines – nor will the objects have constant absolute speed. This can be shown in a fairly simple way.

6. The velocities in the fixed reference frame

A body moving without friction over the Earth's surface will conserve its axial absolute angular momentum because there is no torque around the Earth's rotational axis Ω (Phillips, 2000; Holton, 2004, p. 19; Vallis, 2010, p. 143; Holton and Hakim, 2012, p. 17 f). Specifically, if the oscillation is of the midlatitude 'inertia circle' type, confined between a southerly φ_S and a northerly φ_N latitude, the constant relative velocity V_r is, in the extreme latitudinal positions, directed in straight east or west directions with a relative difference of $2V_r$

By calculating the body's absolute velocity eastwards we will find a mathematically beautiful relationship with the two limiting latitudes φ_S and φ_N. With R_N and R_S for the distances to the Earth's rotational axis we have with conservation of absolute angular momentum, M

$$M = (\Omega R_N + V_r)R_N = (\Omega R_S - V_r)R_S \tag{9a}$$

and for $U_N = \Omega R_N$ and $U_S = \Omega R_S$ for the eastward velocities of points fixed on the Earth's surface at these latitudes

$$(U_N + V_r)\frac{U_N}{\Omega} = (U_S - V_r)\frac{U_S}{\Omega} \tag{9b}$$

which yields the simple relation

$$V_r = (U_S - U_N) \tag{9c}$$

The two latitudes, φ_S and φ_N, between which the inertia oscillation is confined, differ in their absolute (eastward) speed by a quantity that is identical to the relative speed V_r.

The object's absolute eastward velocities in its most poleward (10a) and equatorward (10b) positions can also be written

$$U_N + V_r = U_S \tag{10a}$$

$$U_S - V_r = U_N \tag{10b}$$

which implies that *the absolute eastward velocity of the object in one latitudinal extreme position will equal the absolute velocity of the latitude in the opposite extreme* (Figure 4).

This relation, which to the best of my knowledge, has not been noted before, is a generalization of what Durran (1993, p. 2181) found while studying inertia circle motion close to the North Pole: the same (= zero) absolute velocity both at the poleward extreme (the Pole itself) and at the most equatorward point of the inertia oscillation trajectory. In a paper on inertial motion over the Earth's surface Feussner (1969, p. 263) has provided computational evidence for the relation expressed in Eqs (10a) and (10b).

Therefore, the moving object is changing its velocity in the absolute frame of reference, a real force must be acting. For such a force only the gravitational attraction $\mathbf{g}^*$ can be considered. However, Eq. (6), which describes the motion exactly, does mathematically contain neither the gravitational attraction $\mathbf{g}^*$ nor the centrifugal force $-\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{R})$. This does not mean that these

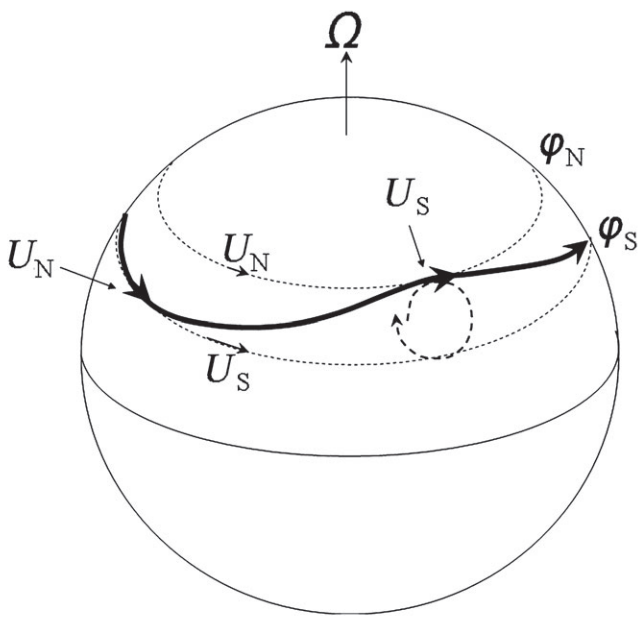

Figure 4. The absolute eastward velocity of an object performing an 'inertia circle' type oscillation between two latitude bands, φ_N and φ_S, where fixed points move eastward with absolute speeds U_N and U_S. The oscillating object's absolute eastward velocity varies between a maximum (= U_S) in its poleward extreme, φ_N, and minimum (= U_N) in its equatorward extreme, φ_S.

forces have vanished physically but that they are not necessary for a *kinematic* description of the motion according to Eq. (6).

Just because the Coriolis deflection can be described kinematically does not mean that the deflection process itself is just a matter of kinematics. To understand the physical mechanism we apply a *dynamical* approach involving the interplay between the dominating forces, Newtonian gravitation and the centrifugal force (Durran, 1993; Ripa, 1997a; Persson, 1998; Phillips, 2000).

7. The physical mechanism responsible for the inertial oscillation

For an object at relative rest on the Earth ($\mathbf{V_r} = 0$), the gravitational component perpendicular to the rotational axis $\mathbf{g}^*_N$, balances the centrifugal force exactly (Figure 5(a)). As is discussed in many textbooks (e.g. Holton, 2004, p. 16 f; Holton and Hakim, 2012, p. 15)[†] an eastward relative motion will make the outward directed centrifugal force stronger than $\mathbf{g}^*_N$ and disrupt the balance, making (apparent) gravity $\mathbf{g}$ no longer directed along the local vertical, but slightly outward (Figure 5(b)). This yields an outward acceleration that, through a simple mathematical analysis, turns out to be $-2\mathbf{\Omega} \times \mathbf{V}_r$ and accounts for the horizontal Coriolis and vertical 'Eötvös effects'. The latter, named after the Hungarian scientist Roland von Eötvös (1848–1919), changes the weight of latitudinal moving objects (Persson, 2005).

For *westward* relative motion the textbooks tend to become rather vague (see e.g. Holton, 2004, p. 16 ff; Holton and Hakim, 2012, p. 16). They could easily just have mirrored the previous reasoning: the westward relative motion will make the outward directed centrifugal force *weaker* than $\mathbf{g}^*_N$ and disrupt the balance, making (apparent) gravity $\mathbf{g}$ no longer directed along the local vertical, but slightly *inward*. This yields an *inward* acceleration that through a simple mathematical analysis turns out to be $-2\mathbf{\Omega} \times \mathbf{V}_r$ and accounts for the Coriolis and Eötvös effects (see Figure 5(c)):

The textbook authors would then have made an interpretation that challenged the conventional view that the Coriolis effect is a purely inertial process and that no real forces play any role. The reason is that the centrifugal force will only be *weakened* by $2\mathbf{\Omega} \times \mathbf{V}_r$, *but still be directed outward and equatorward, not inward and poleward.* The only available physical force that can move anything inwards, towards the poles, is therefore the poleward component of gravitation (true gravity) $\mathbf{g}^*_N$. This was also the conclusion that Durran (1993) reached.

8. The affinity between the Coriolis and centrifugal forces

Another reason for the textbooks' vagueness might be a conception that the Coriolis force is intrinsically different from the centrifugal force just because it is represented by a separate mathematical expression. This can be remedied mathematically by a simple reformulation of Eq. (3a).

$$\frac{d\mathbf{V}_r}{dt} = \mathbf{g}^* - \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{R} + 2\mathbf{V}_r) \qquad (11)$$

where the second term can be regarded as the 'total' centrifugal force through the combined effect of the Earth's rotational velocity and the object's relative velocity.

The view that the 'Coriolis force' is not physically different from the centrifugal force is in line with how Coriolis envisaged 'his' force as an extension or modification to the centrifugal force for motions relative to the rotation (Persson, 1998).

One reason why the physical affinity between the Coriolis and the centrifugal forces has not been evident might be due to the way we conduct the mathematical derivations. Although we use the centrifugal effect for the deflection of zonal motion, the most convenient derivation for the deflection of meridional motion is through angular momentum conservation (Holton, 2004, p. 14 f;

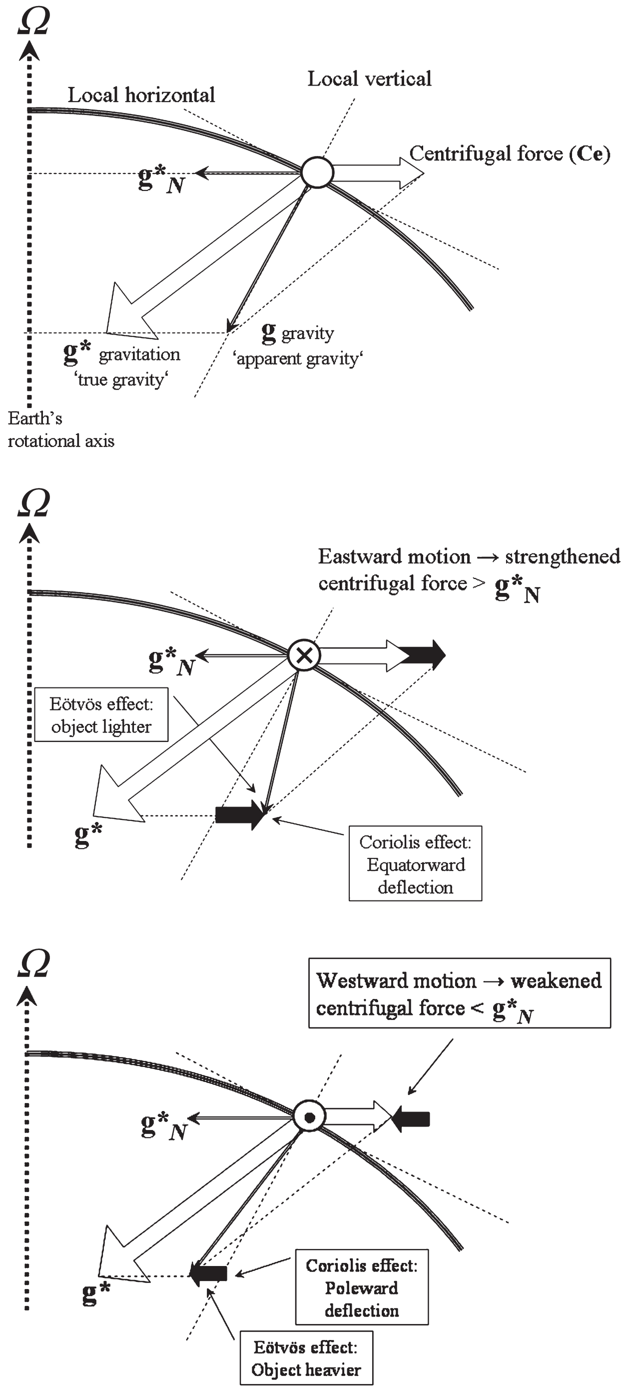

Figure 5. (a) The balances of forces for an object at rest on the surface of a rotating planet. Due to the non-spheroid shape of the planet, only the resultant of the gravitational force and the centrifugal force , the (apparent) gravity vector $\mathbf{g}$, is pointing parallel to the local vertical, not so the gravitational attraction $\mathbf{g}*$, except at the poles and the equator. The gravitational component $\mathbf{g}^*_N$ perpendicular to the rotational axis balances exactly the centrifugal force **Ce**. (b) An eastward relative motion (into the page) makes the outward-directed centrifugal force stronger than $\mathbf{g}^*_N$ and disrupts the balance in (a). This makes $\mathbf{g}$ non-parallel to the local vertical, directed away from the rotational axis. The outward 'extra' centrifugal acceleration (black arrow), perpendicular to the Earth's rotational axis, can be decomposed into one component parallel to the local horizontal, the 'Coriolis effect', and one parallel to the local vertical, the 'Eötvös effect', in this case making eastward-moving objects lighter. (c) A westward relative motion (out from the page) makes the centrifugal force weaker than $\mathbf{g}^*_N$ and disrupts the balance in (a), which makes $\mathbf{g}$ non-parallel to the local vertical. The inward acceleration, *physically caused by* $\mathbf{g}^*_N$ perpendicular to the Earth's rotational axis, can be decomposed into one component, parallel to the local horizontal, the 'Coriolis effect', and one parallel to the local vertical, the 'Eötvös effect', in this case making westward-moving objects heavier.

[†]In an new sentence, inserted in Holton and Hakim (2012, p. 15), it is said that in this case 'axial angular momentum is not conserved'. This is not correct and contradicts what is said elsewhere, e.g. on pp. 17 f, 73 and 77.

Holton and Hakim, 2012, p. 14 f). This might give the impression (as it once did to William Ferrel!) that we are dealing with two different mechanisms that coincidently yield the same result. Phillips (2000, p. 303) suggests a way to remove any ambiguity. A complementary approach is to find a way to derive the Coriolis force also for meridional motion using the centrifugal effect (see Appendix).

9. The Coriolis effect on a rotating parabola

We have seen that motion without friction on the rotating Earth differs fundamentally from the motion over a rotating turntable. However, by *deforming* the turntable into a rotating parabola any motion over its surface will become almost identical to motions over the rotating Earth. This is a common way to demonstrate the Coriolis effect at universities and high schools with a camera rotating with the system. (Klebba and Stommel, 1951; Stommel and Moore, 1989; Durran, 1993; Durran and Domonkos, 1996; Hoskins, 2009).

On a rotating parabolic surface gravity is non-parallel to the local vertical but its resultant with the centrifugal force yields a force parallel to the local vertical (Figure 6). An object at rest relative to the parabola will to an outside observer move around in a circle with a period $\tau = 2\pi/\Omega$. If the object is given an impetus, it will to the outside observer move around in an approximately elliptic path and relative to an observer inside the parabola, perform inertia oscillations with a period $\tau = \pi/\Omega$, just as on the Earth's surface (Figure 7).

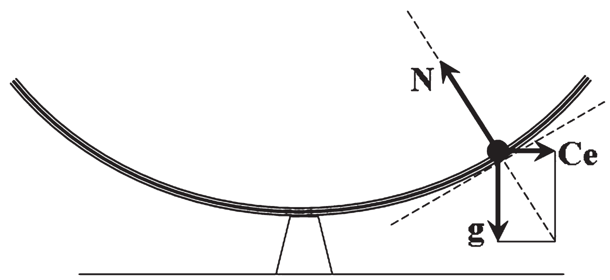

Figure 6. On a rotating parabola the resultant of the centrifugal force (**Ce**) and apparent gravity (**g**) is parallel to the local vertical and balances the reaction force (**N**). An object positioned anywhere on the inner surface will therefore remain at rest. If it is perturbed it will, for an observer inside the parabola, perform oscillatory motions.

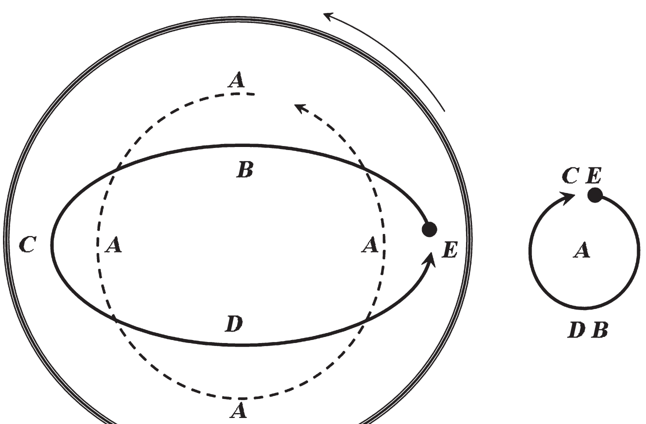

Figure 7. Idealized motion of a body on a counter-clockwise rotating parabolic disc as seen by an outside observer (the left part). The dashed arrow shows the absolute trajectory of the object at relative rest at *A* on the parabola; the solid line arrow shows the absolute trajectory of the object when it has been perturbed. It then moves in an ellipse from *E*, over *B*, *C*, *D* and back to *E*. To the right the same as seen by an observer on the disc. During the time it takes the body to make a full revolution in the fixed frame of reference it makes two clockwise revolutions, *EBC* and *CDE*, in the relative. (After Sverdrup *et al.*, 1942, p. 435; Sverdrup, 1955, pp. 100–104.)

The parabola must be as 'flat' as possible to minimize the effect of the vertical accelerations connected to the object's radial motions. These vertical accelerations will cause its trajectory to gradually drift away in a series of retrogressive loops (Stommel and Moore, 1989, pp. 100, 105) as can be seen in the experiment by Hoskins (2009). The moving object should preferably be something like Durran's and Domonkos's *sliding* cylinder rather than a *rolling* ball, due to the complications arising from the ball's own angular momentum conservation.

10. The role of friction in the Coriolis effect

In their experiment Durran and Domonkos (1996) realized that if there is no frictional coupling between the object and the underlying parabolic surface, it would not matter whether or not the parabolic dish was rotating. They could therefore conceivably economize by driving only the unit containing the rotating-frame camera.

The object was launched along the *non-rotating* parabola's wall with a tangential speed identical to the previous rotation. An outside, stationary observer would still see the object go around in a circular path, an analogue to some amusement park motor cycle races courses (Stommel and Moore, 1989, p. 97). From the moving camera perspective it would, however, be seen resting motionless. By perturbing the motion, e.g. by launching the object with a slightly different speed, it will to the outside observer appear again as moving in an approximately elliptic path, to the rotating camera it would appear to perform inertia oscillations, identical to the ones in Figure 7.

The Durran and Domonkos (1996) experiment thus shows that it is not through friction that the object 'knows' it is in a rotating system, but through the shape of the underlying surface defined by the strength and direction of the reaction force **N** in Figure 6 (Phillips, 2014).

Removing the need to have an underlying rotating surface leads us into celestial mechanics where we find Coriolis effects in unexpected circumstances. Butikov (2001) shows mathematically how an astronaut in an orbiting satellite should dispose of an unwelcome space alien so that the Coriolis force does not return him to the satellite. Greenberg and Davis (1978) show how Coriolis forces resulting from the interacting of one centrifugal force and *two* gravitational attractions make hundreds of asteroids cluster in Jupiter's 'Greek' and 'Trojan' Lagrange points L4 and L5.

11. The role of rotation in the Coriolis effect

Durran and Domonkos (1996) did not discuss whether their result had any relevance for the Earth. We can make the thought experiment that the Earth were to stop rotating, while keeping its non-spherical oblate ellipsoid form. We then launch an object without friction 'eastward' along a certain (previous) latitude, with the same speed as the (once moving) latitude. For a hypothetical observer in space, travelling with the object, it would appear to be at rest. If the object's velocity was perturbed by V_r a hypothetical observer in space, travelling with the body, would see it perform inertia oscillations just as a stationary observer on the Earth when the earth is rotating.

Figure 8(a) depicts schematically the absolute and relative motions for an object released with a relative meridional northward velocity of 10 m s^{-1} at the latitude of Oxford, 51.75° where the Earth's rotation eastward is 288 m s^{-1}. The object's absolute motion will oscillate between 283 and 293 m s^{-1} and, seen by a hypothetical observer fixed in space, and in line with Figures 1 and 4, be a curtate cycloid trajectory confined between two latitudes, in this case between 51° and 52.5°. A local observer in Oxford would see the object perform an 'inertia circle' oscillation with a radius of curvature around 8 km (small arrows).

In Figure 8(b) the earth is no longer rotating but maintains its non-spherical form. An object is launched 'eastwards' at 288 m s^{-1}, the same speed as the once moving Earth at that

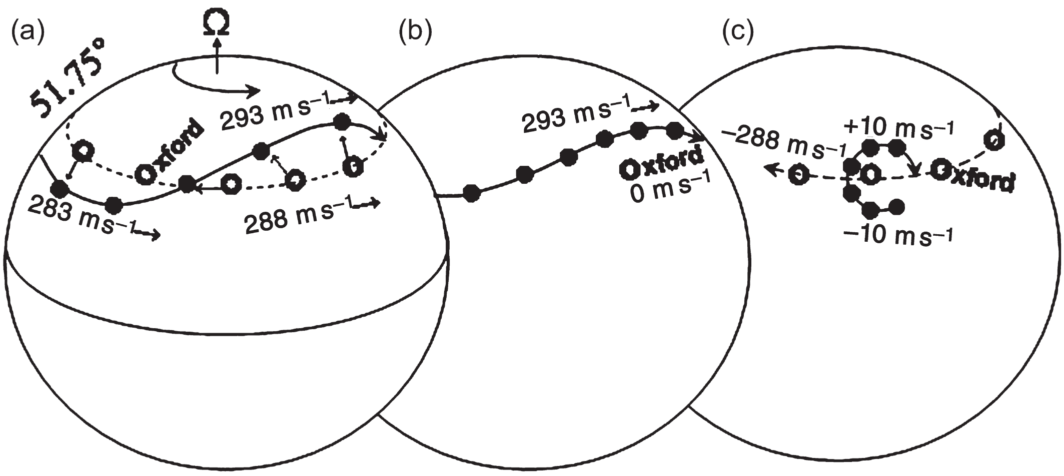

Figure 8. Absolute and relative motion without friction of an object performing inertia oscillations in the midlatitudes (in this case at the latitude of Oxford 51.75°) over a rotating and non-rotating ellipsoid shaped Earth (see text for details). (a) Rotating Earth seen from an outside, fixed position, (b) and (c) Non-rotating, but still ellipsoid, Earth seen from an outside, fixed position and 'eastward' moving, perspective.

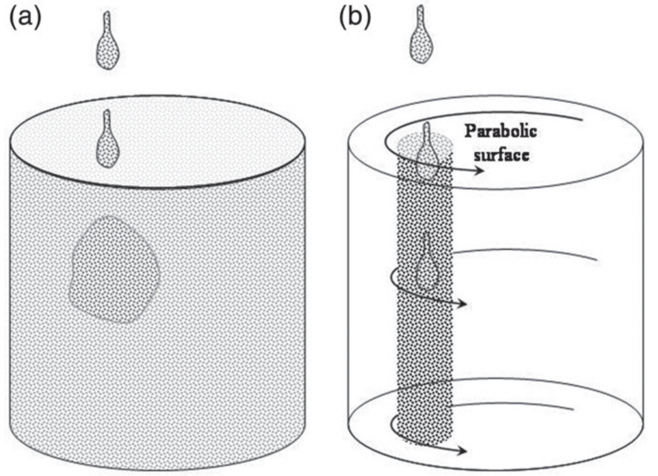

Figure 9. Schematic images of the formation of Taylor–Proudman columns in a rotating water tank. (a) Ink inserted into the water of a *non-rotating* tank will spread over the whole water volume and colour it uniformly. (b) The tank has been brought into rotation and the (new fresh) water has settled in a way that makes it rotate with the tank as a solid unity. When the ink is dropped into the water (in the same relative position following the rotation) it will sink and not disperse, instead forming vertical coloured columns. The upper surface has through the rotation formed a parabolic surface that provides an inward directed horizontal pressure-gradient force, which balances the inward directed centrifugal force.

latitude, plus an initial meridional component of 10 m s^{-1}. An observer fixed in space, or in Oxford, sees the same absolute trajectory as in the 'normal' case, oscillating between 293 m s^{-1} at latitude 52.5° and 283 m s^{-1} at 51°.

In the right-hand image (Figure 8(c)) nothing has changed for the local stationary Oxford observer, but the hypothetical space observer is now following the object 'eastwards' around the non-rotating Earth. Beneath him he can see Oxford moving in the opposite 'westward' direction at the same speed. The launched object, however, will appear to him perform 'inertia circle' oscillations with a radius of about 87 km.

This thought experiment, inspired by Durran and Domenkos's (1996), is an other way to illustrate that there does not have to be an underlying, rotating surface to make Coriolis effects possible. A more controversial interpretation, that the deflecting force is primarily not caused by the rotation, but by the Earth's non-spherical figure and would be just as effective on a non-rotating but still non-spherical earth, was made by the German geophysicist Karl Feussner at a conference in Berlin in 1968 (Feussner, 1969). That the rotation of the surface is secondary and all that is needed is a parabolic form, was also emphasised by the Mexican oceanographer Pedro Ripa in his book on the Coriolis force (Ripa, 1997b, Chapter VIII).

Durran and Domenkos (1996) never drew these conclusions but they were perhaps implicit when they questioned if their commercially easily available parabolic disc was the most natural equipment for their experiments? However, as will be seen below, in their further discussion they pointed out the decisive role of rotation in *creating* the non-spherical form.

12. The common denominator – the balancing of the centrifugal force

Durran and Domonkos's experiments were made in collaboration with Professor Emeritus Norman A. Phillips. In the 1950s, then at the Massachusetts Institute of Technology, Phillips had created a parabolic surface by rotating a disc of slightly liquid cement until the cement hardened after a night's running. It was then polished to provide a very smooth surface.

Durran and Domonkos (1996) commented that from a pedagogical viewpoint, it would have been nice to follow Phillips's original strategy because his construction provides an experimental confirmation that the free surface of a rotating fluid *must* deform into a parabolic surface in order to be in equilibrium between true gravity and the centrifugal force. In the same way the rotation of the Earth deforms it into a non-spherical shape.

This kind of cancellation of the centrifugal force by a 'natural' creation of an inward component of a real force occurs not only with the rotating parabola or the rotating Earth but also in G.I. Taylor's famous water-tank experiments (Figure 9). The rotation creates in the fluid an upper parabolic surface where an inward pressure-gradient force balances the outward-directed centrifugal force (Batchelor, 1967, pp. 557–559, Batchelor, 1994; Taylor, 1974).

The unbalanced part of the centrifugal force, the Coriolis force, will make any ink particle with a horizontal velocity component, i.e. a component perpendicular to the rotational axis, return in a small inertia circle. With one revolution in 2 s any ink particle 'trying to escape' horizontally with a velocity of 6 mm s^{-1} will be brought back in an inertia circle motion confined within a diameter of 4 mm. (With this information it is left to the reader to figure out why a pingpong ball, released from the bottom of a water-filled tank, will rise more slowly through the water if the tank is rotating.)

G. I. Taylor's famous experiment shows, thanks to the colouring of the water, how the *mechanical properties* of a liquid or gas change due to the rotation, how the rotation makes them 'elastic', 'rigid', 'stiff' or 'obstructive' (Prandtl, 1952, p. 355; Baker, 1966, Batchelor, 1967, pp. 556–557; Cushman-Roisin, 1994, pp. 4, 131; Persson, 2001; Vallis, 2010, p. 88). In the same way the mechanical properties of the atmosphere and oceans will change due to the Earth's rotation, or as formulated by Vallis, 2010, p. 57): *'If the solid Earth did not bulge at the equator, the* ***behaviour*** *of the atmosphere and oceans would differ significantly from that of the present system'*. This is something that is not easily conceptualized from the 'illusory' Coriolis effects demonstrated on a turntable

13. Three misinterpretations of the Coriolis effect

There are several pedagogical ways to approach the Coriolis effect that give rise to misleading notions: (i) that it can be fully understood through the turntable analogy; (ii) that the shape of the Earth has no relevance for the terrestrial Coriolis effect; (iii) that it, when all comes about, is just 'illusory'.

13.1. *Misleading justifications for the turntable model*

Although Holton (2004, p. 14 f), Durran and Domonkos (1996, p. 557) and others have pointed out that the standard derivation of the Coriolis effect, physically applicable on a turntable, potentially can be misleading for geophysical interpretations, it is still a common approach.

13.1.1. *The f-plane approximation*

Durran (1993, p. 2183) suggests that one source of the confusion is to interpret the f-plane physically, as if it indeed were a real turntable. On an f-plane the motions are *prescribed* to be determined only by a constant Coriolis term $2\mathbf{\Omega} \times V_r$, with any centrifugal force intentionally ignored. This f-plane approximation is, however, a good physical approximation only very close to the centre of rotation of a real turntable. For a body moving with 1 m s^{-1} on a turntable with one revolution in 2 s (and thus $\Omega = \pi$), the centrifugal force is smaller than the Coriolis force only 0.2 m from the centre of rotation.

13.1.2. *The 'Hadley Principle'*

Applying the turntable logic directly to the rotating Earth is the basis for 'Hadley's Principle' from 1735 (Persson, 2009). This intuitively appealing, but *totally* erroneous, explanation has not only deceived generations of students but also some great minds such as Arnold Sommerfeldt and Max Born. The distinguished oceanographer Adrian Gill strongly promoted Hadley's reasoning in his textbook and claimed that it was inspired by conservation of angular momentum (Gill, 1982, pp. 23, 189, 369, 506 and 549). Hadley assumed conservation of *absolute velocity* and could not possible have thought in terms of angular momentum, because it was only later in the 1700s that physicists realized that Kepler's second Law (= conservation of angular momentum) could also be used in terrestrial mechanics.

13.1.3. *The 'simplified' derivation*

A simple and mathematically impeccable, but physically misleading, derivation of the $2\Omega V_r$ term was presented in the 1840s by the French mathematician Joseph Bertrand (Persson, 2005). It has since then gained widespread popularity, even in some academic textbooks, as a complement to the rigorous vector derivation. It makes use of two erroneous assumptions: Hadley's idea about conservation of absolute velocity for motions over the Earth and that the deflection on a flat turntable is solely due to the Coriolis force. The first assumption overestimates the Coriolis effect, the second underestimates it, and because these errors cancel the derivation yields the 'right' result (Figure 10):

However, conservation of absolute velocity is a justified condition on a flat turntable, *but not on a rotating planet*, whereas the condition that the Coriolis force is solely responsible for the deflection is justified on a rotating planet, *but not on a turntable*. Bertrand's 'simplified' derivation is interesting from a philosophical point because it shows that *it is possible to obtain correct results from erroneous assumptions even if the mathematical treatment is correct.*

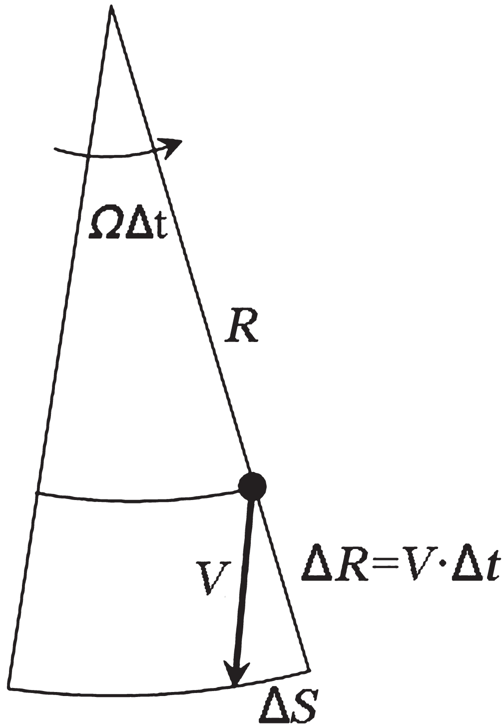

Figure 10. The deceptive 'simplified' derivation of the Coriolis force: an object is moving outward in a rotating system with relative velocity V. During Δt it covers the distance $\Delta R = V \cdot \Delta t$ during which time the system rotates anticlockwise by an angle $\Omega \cdot \Delta t$ and causes a clockwise deflection by $\Delta S = \Omega \cdot \Delta t \cdot V \cdot \Delta t$, which according to the relation $\Delta S = a \cdot (\Delta t)^2/2$ yields the acceleration $a = 2\Omega \cdot V$.

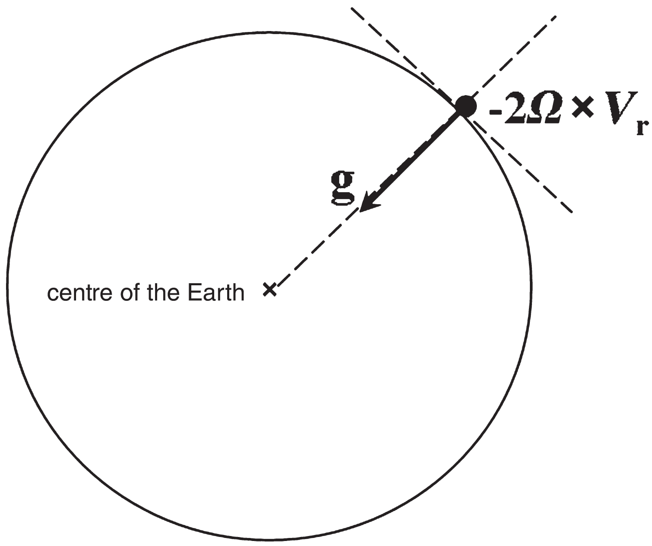

Figure 11. The mathematical model for numerical modelling: the Earth is considered a perfect rotating sphere with no centrifugal force and therefore 'apparent gravity' **g** will be identical to only 'true gravity' **g***, which for a spherical planet will point straight toward the centre with no horizontal components. Any relative motion V_r on the Earth's surface is assumed to be affected horizontally only by the Coriolis term $-2\mathbf{\Omega} \times V_r$.

13.2. *Poor interpretation of the equations of motion*

A correct mathematical derivation is a necessary but not sufficient condition to obtain a qualitative 'feel' for the physical processes or mechanisms involved in the Coriolis effect.

13.2.1. *The spherical Earth approximation*

The spherical Earth model is a useful mathematical model, for example in numerical weather prediction, but it is nevertheless unphysical (Gerkema *et al.*, 2008). Leaving aside the fact we do not know any rotating planet that is spherical, any motion without friction over such a spherical planet would physically be great circles in an inertial frame of reference and yield more complex loops in the non-inertial (McIntyre, 2000; Earley, 2013).

The main reason for this approximation is therefore *not*, as is often said, that the Earth is 'almost a sphere', but that we can represent the physical real non-spherical Earth with an unphysical mathematical model where the motions will be the same. To achieve this, however, we must also make other unphysical simplifications, such as ignoring the centrifugal effect of the rotation that makes the gravitational attraction everywhere parallel to the local vertical (Figure 11). Although it physically should be uniform on a spherical Earth it can, as James (1994, p. 9) points out, for modelling purposes, unphysically, be prescribed as a function of latitude. There is an excellent exposure in Vallis (2010, pp. 55–57) on these questions.

13.2.2. *The 'metric terms'*

So-called 'metric terms' u^2/R and uv/R appear when we derive the deflection of a (west–east) latitudinal motion using the centrifugal force approach and deflection of a meridional (north–south) motion using conservation of angular momentum. These terms are rightly neglected, but often for the wrong reason. It is true that they are small for 'synoptic-scale motions' or when the speed $\ll \Omega R$, the rotation of the Earth (Holton, 2004, p. 16 f; Gerkema *et al.*, 2008; Holton and Hakim, 2012, p. 15f)[‡]. The main argument, however, is *physical*: the 'metric terms' result from a *constrained* motion, prescribed to be along latitudes and longitudes respectively. Even on a non-rotating Earth a train between Reading and London would, equally in both directions, physically experience a southward acceleration against the rails, although much weaker than the Coriolis force (for the rotating case). The motion of the atmosphere and oceans. however, is not constrained in this way. This makes these 'metric terms' physically irrelevant, or else Eq. (5) would be an approximation. For their roles in the 'primitive equations' see Vallis (2010, p. 61).

13.2.3. *The rotational period of the Earth*

In determining the value of the Coriolis parameter $-2\Omega V_r \sin\varphi$ the rotational period Ω should be calculated on the sidereal day of 23 h and 56 min, which corresponds to the Earth's rotation versus the fixed stars. There seems, however, to be a general unawareness that using the sidereal day is the same as taking into account the small Coriolis contribution from the orbiting around the Sun. Some textbooks (e.g. Godske *et al.*, 1957, p. 233) even deny that this is the case. Experiments run at the European Centre for Medium-range Weather Forecasts showed clear differences after about 5 days.

Consequently, considering the rotation rate of one of Jupiter's and Saturn's gaseous moons against the fixed stars will implicitly take into account the Coriolis effect due to (i) the moon's rotation around its own axis, (ii) its rotation around the mother planet and (iii) the moon's rotation around the Sun. The rotation of the solar system around the galaxy can, however, be neglected!

[‡]With these scale arguments Holton (2004) and Holton and Hakim (2012) contradict themselves on the following pages, when they calculate the deflection of a rapidly eastward moving ballistic missile and (correctly) do not take into account any 'metric terms', which would in such a case have more than doubled the deflection.

13.3. *Misrepresentation of mechanical definitions related to the Coriolis effect*

Misinterpretations also occur when stringent definitions in classic mechanics are interpreted colloquially.

13.3.1. *Definition of 'work'*

Statements that the Coriolis force is a 'fictitious' or 'apparent' force that 'cannot do work' does not mean that it is 'doing nothing'. The stringent mechanical definition of 'work' is the scalar product between a force and the displacement of the body it is acting on, which is associated with a conversion between potential and kinetic energy. 'The Coriolis force is doing no work' means simply that it, always perpendicular to the motion, can never contribute to any conversion between kinetic and potential energy. For 'negative work' see (Persson, 1998, pp. 1379, 1382 ff).

13.3.2. *Are 'inertia oscillations' really inertial?*

Durran (1993) questioned the use of words such as 'inertia', 'inertial motion' and 'inertial oscillation' because a real force, gravitation, is involved. This has caused some controversy (Ripa, 1997a; Early, 2012, among others). An argument in support of Durran is the common (mis)conceptions surrounding the Foucault pendulum. Its oscillation is often explained as a motion without friction under inertia, with the plane of the swing steadily pointing to a specific fixed star, while the Earth is rotating under it. Generations of students have struggled to accommodate this 'easily understood' explanation, which suggests a uniform period (of one sidereal day) everywhere on the globe, while the mathematics (and observations!) yield a period of the sidereal day *divided by sine of the latitude.* As shown by Phillips (2001, 2004) the Foucault pendulum motion is *not* inertial because it is highly affected by the Earth's gravitational force, which makes the plane of swing slowly precess versus the fixed stars, except at the poles.

14. From Charles Delaunay 1859 to Dale Durran 1993

In retrospect, Durran would have preferred the title 'Is the Coriolis Force *Alone* Really Responsible for the Inertia Oscillation?' (D. Durran and N. Phillips, 2000; personal communication). It would perhaps have expressed more strongly his concern that the $2\mathbf{\Omega} \times V_r$ term was a kinematic description only of *how* the oscillation performed, not *why*. To accomplish this he made a dynamic analysis which gave the result that the gravitational attraction, a very real force, was highly instrumental in the 'inertial' oscillation.

Durran's analysis, reproduced and extended here, might appear new and controversial, but more or less identical discussions have been conducted over the past 150 years, the first being in 1859 by the astronomer Charles Eugène Delaunay (1816–1872) during a session in the French Academy (Persson, 2005). In 1933, halfway between Delaunay and Durran, we find in *Physikalische Hydrodynamik*, a standard work by the pillars of the legendary Bergen School, a long and thorough mathematical and physical analysis of the effects of the Earth's rotation on an object moving without friction (Bjerknes *et al.*, 1933, pp. 453–473): on page 459 Durran's discussion of an object conducting an 'inertial oscillation' is more or less replicated.

15. Discussion

Although today there is a broad understanding of the basics of the relativity theory and quantum mechanics, why is the Coriolis effect still a mystery? Symptomatically Richard Feynman, who eloquently managed to explain almost everything in physics but expressly stayed away from meteorology, regarded the analysis of rotating fluids as unsolvable (Feynman *et al.*, 1977, pp. 3–7 to 3–9) and failed to explain the Coriolis effect (Feynman *et al.*, 1977, p. 19-8; Tiersten and Soodak, 1998).

The author has elsewhere (Persson, 2010) suggested three factors that complicate attempts to conceptualize the Coriolis effect: its counterintuitive nature, the formalistic way it is taught and an unawareness of the distinction between mathematics and physics.

1. Many processes in classic mechanics, of which dynamic meteorology is a branch, are quite counterintuitive. Most of us are since childhood familiar with the Coriolis effect on a carousel, but only scientists in boundary-layer meteorology, oceanography and fluid dynamics have direct experience of how it manifests itself in the form of 'inertial oscillations' (Thorpe and Guymer, 1977; van de Wiel *et al.*, 2012). This might be the reason why textbooks in these subjects often provide better interpretations of the Coriolis effect than other meteorological textbooks.
2. A popular criticism of dynamic meteorology education is that it is 'too mathematical'. Someone who might have agreed, although not for the popular reasons, was the late Pierre Gilles de Gennes (1932–2007) the 1991 Nobel Prize Laureate in Physics. In an educational debate in France he held that mathematics is the easiest part of physics (de Gennes and Badoz, 1996). The difficulty, according to de Gennes, lies in the *interpretation of the mathematics*, how it relates to observations and how it connects with other theories. The Coriolis effect is a striking example of his thesis: the correct mathematical derivation was first made by Laplace and Gauss more than 200 years ago – and we are still debating what it physically means.
3. Making a distinction between mathematics and physics enables us to understand, and make use of, the ability of mathematics to describe a certain physical process by different formalisms. As pointed out by Gerkema *et al.* (2008), it is, however, always important to find out to what extent mathematical simplifications or conditions are physically valid. The spherical Earth model (section 13.2.1) yields accurate descriptions (and forecasts!) of the atmospheric flow, but is not appropriate as a basis for a physical understanding.

These three factors are represented in George Hadley's popular but highly misleading 'Principle': (i) air parcels can never move in the intuitively appealing way Hadley suggested (see section 13.1.2), but rather along quite counterintuitive trajectories; (ii) a mathematical derivation, partly based on Hadley's erroneous model, can still yield the 'right' solution (see section 13.1.3); (iii) when Hadley's erroneous condition of conservation of the absolute velocity is replaced by the correct condition of conservation of angular momentum this yields even more excessive winds, because the change is done with wrongly specified physical conditions (paper under preparation).

16. Summary

Although relative motion V_r without friction, both over a turntable and a rotating planet, involves a deflecting mechanism, expressed by the term $-2\mathbf{\Omega} \times \mathbf{V}_r$, the two scenarios differ fundamentally physically. This is because of the presence of an unbalanced centrifugal effect in the former scenario but not in the latter.

A proper intuitive understanding of the deflection over a rotating planet, on a rotating parabola or in a rotating water tank, might require more mental effort than a scant reference to the 'illusory' deflection over a turntable, but the effort is rewarded because it makes it intuitively easier to understand the fundamental physical effects that the rotation has on gaseous or liquid substances, making them 'rigid' or 'stiff', resisting external forces.

The general circulation of the atmosphere, with its jet streams, Rossby waves and vortices, will with this interpretation appear as manifestations of an eternal 'combat' between two opposing forces: *the pressure-gradient force trying to equalize horizontal contrasts, and the Coriolis force trying to restore them.*

Acknowledgements

Comments and criticism from discerning reviewers have greatly improved this paper from its original version. It was during discussions with Professor Emeritus Norman A. Phillips more than 15 years ago that the significance of Dale Durran's two papers became clear to me.

Appendix

A.1. Derivation of the Coriolis force from a centrifugal perspective

To clarify that the Coriolis force and the centrifugal force are not two intrinsically different types of forces, the former is here derived entirely from a centrifugal, or rather centripetal perspective. It is based on an idea by Jones and Wallingford (1969), who seem to have been inspired by a similar approach in Newton's 'Principia'. The derivation is carried out in an absolute frame of references, thus avoiding the problem of accommodating both *absolute* motion (the path of the moving object) and *relative* accelerations (e.g. the centrifugal acceleration) in the same image.

Note that the Coriolis acceleration $+2\Omega V_r$ is not the 'fictitious' Coriolis force for unit mass, but an acceleration due to a *real* force and is directed to the *left* for counter-clockwise rotation (like the Northern Hemisphere). On a moving body it counters the Coriolis force, keeping a body in a straight relative motion (see Vallis, 2010, p. 53 and footnote 1, p. 115).

A.1.1. Derivation of the centripetal acceleration

Jones and Wallingford (1969) considered an object that under inertia should have moved rectilinearly from A to B. Fixed in a rotating system the body is brought, during time Δt, by an inward centripetal acceleration, a, into a circular trajectory to C with a deflection ΔS (Figure A1(a)). Using the Pythagorean theorem

$$(R + \Delta S)^2 = R^2 + (\Omega \cdot R \cdot \Delta t)^2, \qquad \text{(A1)}$$

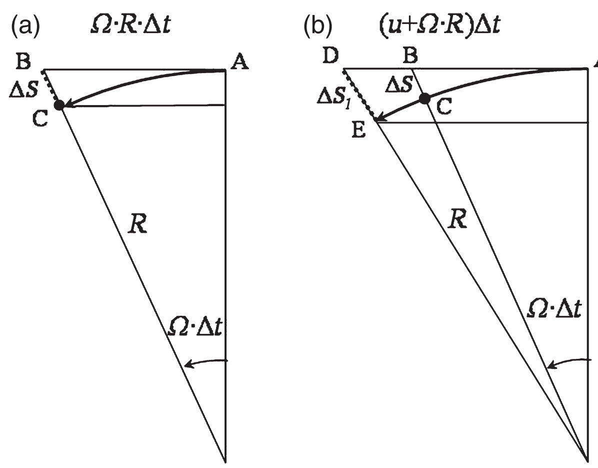

Figure A1. (a) The derivation of the centripetal acceleration for a body fixed in the rotating system. (b) The derivation of the centripetal and Coriolis accelerations for a body moving tangentially relative to the rotating system. See text for further details.

which, ignoring non-temporal quadratic terms, yields

$$2\Delta S = \Omega^2 \cdot R \cdot (\Delta t)^2, \tag{A2}$$

and with the familiar relation $2\Delta S = a \cdot \Delta^2 t$ yields the equation $a = R \cdot \Omega^2$ for the centripetal acceleration.

A.1.2. Derivation of the Coriolis acceleration for tangential relative motion

An object in the same anticlockwise rotating system as above has a tangential component (u) relative to the rotation. Instead of moving from A rectilinearly over B towards D the object is brought, during time Δt, due to an inward centripetal acceleration, a, into a circular trajectory to E with a deflection ΔS_1 (Figure A1(b)).

Again applying the Pythagorean theorem

$$(R + \Delta S_1)^2 = [(u + R \cdot \Omega) \cdot \Delta t]^2 + R^2, \tag{A3}$$

and with the same arguments as above we have

$$2\Delta S_1 = \Omega^2 \cdot R \cdot (\Delta t)^2 + 2\Omega \cdot u \cdot (\Delta t)^2 + u^2 \cdot (\Delta t)^2/R, \tag{A4}$$

which, again using $2\Delta S = a \cdot \Delta^2$t, yields three acceleration terms: $\Omega^2 \cdot R$ (the centripetal acceleration), $2\Omega \cdot u$ (the Coriolis acceleration) and u^2/R (the 'metric' term), as discussed above in section 13.2.2.

A.1.3. Deriving the Coriolis acceleration for radial motion

An object in the same anticlockwise rotating system has a radial component (v) relative to the rotation. Instead of moving from A rectilinearly towards B the object is brought, during time Δt, into a curved trajectory to G with a deflection ΔS_2 (Figure A2(b)). The deflection from B to C is due to the above-mentioned centripetal acceleration $\Omega^2 \cdot R$, the deflection from C to G by a combination of the radial translation ($v \cdot \Delta t = \Delta R$) and a tangential acceleration, a, which over time Δt carries the object over distance ΔS_2 (Figure A2(b))

Had the object been stationary in the rotating system it would, as stated above, have reached C after covering the distance $R \cdot \Omega \cdot \Delta t$. Because the tangential acceleration is directed against the rotation, the object will, when it reaches G, only have covered the distance $(R - \Delta R) \cdot \Omega \cdot \Delta t$.

For $\Omega \cdot \Delta t \ll 1$ the difference between the distances $\Delta S_2 \approx \Delta R \cdot \Omega \cdot \Delta t = v \cdot \Omega \cdot (\Delta t)^2$, which through $2\Delta S = a \cdot \Delta^2 t$ yields the tangential acceleration $a = 2\Omega \cdot v$, the Coriolis acceleration. The metric term uv/R that results from the derivation using angular momentum (Holton, 2004, p. 16; Holton and Hakim, 2012, p. 15) is absent here because $u = 0$ for meridional v motion. It may be conjectured that any correct derivation of the Coriolis force, mathematically or geometrically, must be made in conjunction with the derivation of the centrifugal force.

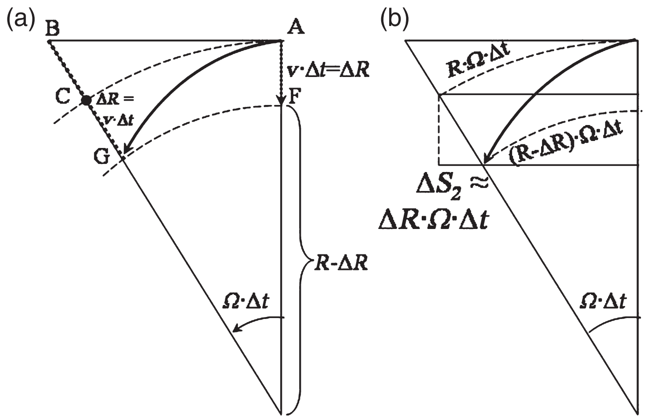

Figure A2. The derivation of the Coriolis acceleration for a radially inward moving object. (a) Highlights the outline of the motion and (b) provides additional mathematical details. See text for further details of the derivation.

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